MATH 32 FALL 2012
MIDTERM 1 - SOLUTIONS

(1) (6 points) Find all values of x satisfying the inequality

$$\frac{x+2}{x-1} < 2$$

Solution: We'll multiply both sides by $x - 1$.
Case 1: $x - 1 \geq 0$. This happens when $x \geq 1$. Then we have

$$\begin{aligned} x + 2 &< 2(x-1) \\ x &< 2x - 2 - 2 \\ -x &< -4 \\ x &> 4 \end{aligned}$$

Case 2: $x - 1 < 0$. This happens when $x < 1$. Then we have

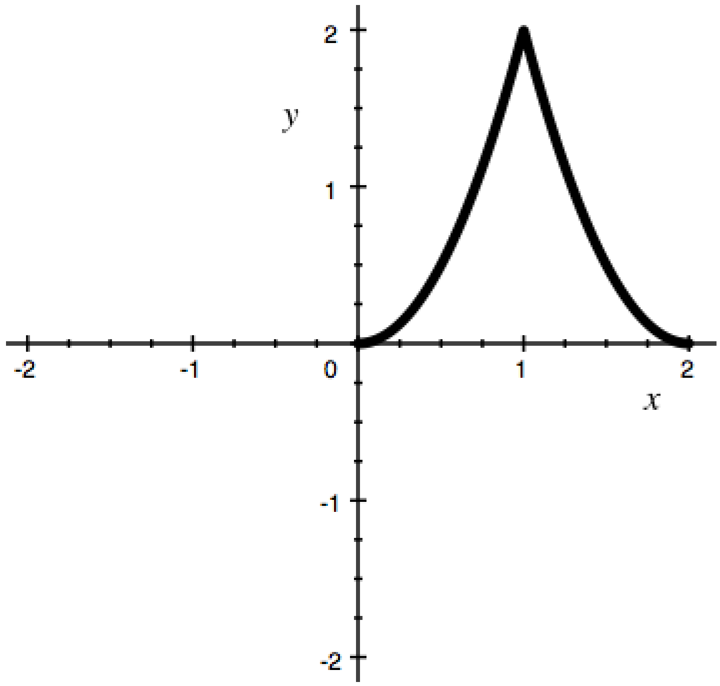
$$\begin{aligned} x + 2 &> 2(x-1) \\ x &> 2x - 2 - 2 \\ -x &> -4 \\ x &< 4 \end{aligned}$$

So when $x \geq 1$, the solutions are all $x > 4$. When $x < 1$, the solutions are all $x < 4$. Putting these together, the solutions are $(-\infty, 1) \cup (4, \infty)$.

(2) (12 points) Let $f(x) = x^2 - 4x + 6$. The graph of f is a parabola. Find an equation for the line containing the vertex of this parabola and its y-intercept. For partial credit, make sure to clearly write down the vertex and y-intercept once you have found them.

Solution: To find the vertex, complete the square. $f(x) = (x-2)^2 - 4 + 6 = (x-2)^2 + 2$. So the vertex is $(2, 2)$.
To find the y-intercept, plug in 0. $f(0) = 0^2 - 4 \cdot 0 + 6 = 6$. So the y-intercept is $(0, 6)$.
The slope of the line containing $(2, 2)$ and $(0, 6)$ is $\frac{6-2}{0-2} = \frac{4}{-2} = -2$.
In point-slope form, the line is given by $y - 6 = -2(x - 0)$, or $y = -2x + 6$.

(3) Let f be the function whose graph is pictured below:

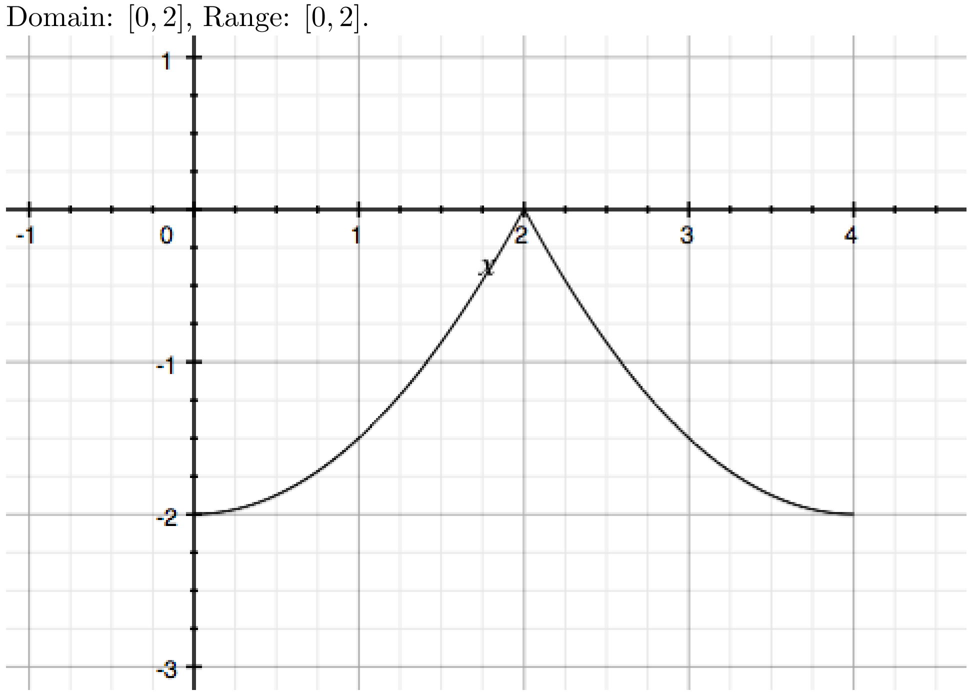


(a) (6 points) Inferring from the picture, what are the domain and range of f?
(b) (6 points) Sketch a graph of the function

$$g(x) = f\left(\frac{x}{2}\right) - 2.$$

Be sure to clearly label your axes.

Solution:
(a) Domain: $[0, 2]$, Range: $[0, 2]$.



(b)

(4) Let $f(x) = \sqrt{x-1}$ and $g(x) = x^3 - x^2 - 2x + 1$.
(a) (6 points) What is the domain of the composition $g \circ f$?

(b) (6 points) Find a formula for the composition $(f \circ g)(x)$.
(c) (6 points) What is the domain of $f \circ g$?

Solution:

(a) The domain of g is all real numbers, so the only problems come from f. The domain of f is $[1, \infty)$, so the domain of $g \circ f$ is also $[1, \infty)$.
(b) $(f \circ g)(x) = \sqrt{(x^3 - x^2 - 2x + 1) - 1} = \sqrt{x^3 - x^2 - 2x}$.
(c) The domain is all values of x which do not result in taking the square root of a negative number, that is, all x such that $x^3 - x^2 - 2x \geq 0$.
Factoring this polynomial, we want $x(x^2 - x - 2) = x(x-2)(x+1) > 0$.

	$(-\infty, -1)$	$(-1, 0)$	$(0, 2)$	$(2, \infty)$
x	$-$	$-$	$+$	$+$
$(x-2)$	$-$	$-$	$-$	$+$
$(x+1)$	$-$	$+$	$+$	$+$
Total:	$-$	$+$	$-$	$+$

So the domain is $(-1, 0) \cup (2, \infty)$.

(5) (6 points) Write $(2x^2)^{-2} - 2(x^2)^{-2}$ as a single fraction.

Solution:

$$\frac{1}{(2x^2)^2} - \frac{2}{(x^2)^2} = \frac{1}{4x^4} - \frac{2}{x^4} = \frac{1}{4x^4} - \frac{8}{4x^4} = \frac{-7}{4x^4}$$

(6) (6 points) Give an example of a polynomial of degree 5 which has zeros 0, 2, and 4, and no other zeros. You may write the answer in factored form.

Solution: $x^3(x-2)(x-4)$.